Abstract
In this work we describe a method to handle curved orbits in wavenumber domain focusing algorithm for high-resolution SAR data acquired by Low Earth Orbit satellites using spotlight mode. The standard wavenumber domain focusing algorithm make assumptions that start to be invalid when applied to high-resolution spotlight SAR data acquired in spaceborne low Earth orbit (LEO) configurations. The assumption of a hyperbolic range history is no longer accurate for sub-metric spatial resolutions due to the satellite curved orbit. The proposed method is used to estimate the satellite velocity and closest-approach range distance of the rectified effective orbit which minimizes the phase errors over the whole scene coverage. This allows to use all frequency domain focusing kernels developed to focus SAR images acquired with a stripmap mode. We use this method with the $\omega$-k algorithm, and its approximation Fast $\omega$-k (F$\omega$-k), demonstrating its effectiveness to focus COSMO-SkyMed (CSK) SAR images obtaining, respectively, sub-metric and metric azimuth spatial resolution.

Keywords: SAR processing, spotlight mode, COSMO-SkyMed.

Introduction
Spatial resolution is a key parameter in several remote sensing applications like urban mapping or post-disaster damage assessment. Optical images, which is commercially available with resolution of up to 50 cm nowadays, provides a reliable tool in several situations except during bad weather conditions or at night. The new high-resolution synthetic aperture radar (SAR) satellite systems, like COSMO-SkyMed and TerraSAR-X, offer an alternative in these cases. In the SAR standard acquisition mode (called stripmap), the direction to which the antenna points is held constant as the SAR platform moves on, allowing an azimuth resolution of up to half of the antenna dimension [Cumming and Wong, 2004]. In spotlight mode [Carrara et al., 1995], the azimuth beam always illuminates the
the same area on the ground (staring mode) by steering the radar antenna, and a finer azimuth resolution can be achieved. Sliding spotlight mode [Belcher and Baker, 1996; Torre and Capece, 2011; Zamparelli et al., 2012] is a well-known novel imaging mode which can obtain better azimuth resolution than stripmap mode and greater azimuth coverage than spotlight staring mode. Spotlight mode leads to greater azimuth bandwidth than stripmap mode does [Prati et al., 1991]. However, to relieve the range ambiguity problem and the burden of large amounts of data, the Pulse Repetition Frequency (PRF) is generally smaller than the azimuth bandwidth in spaceborne spotlight SAR. As a result, the azimuth spectrum folding phenomenon occurs. This phenomenon makes it impossible to apply the frequency domain imaging algorithms for stripmap mode to process spotlight data directly. Different kinds of solutions have been developed to overcome the spectrum folding problem: the azimuth oversampling technique [Prati et al., 1991; Prati and Rocca, 1992; Belcher and Baker, 1996], the sub-aperture technique [Mittermayer et al., 2003], and the deramping-based technique [Lanari et al., 1993, 2001]. After azimuth spectrum unfolding, traditional stripmap mode imaging algorithms can be used to obtain focused images. Different frequency domain focusing algorithms have been proposed to obtain a good accuracy/efficiency tradeoff. The most popular algorithms are the range-Doppler (RD) algorithm [Smith, 1991; Carrara et al., 1995; Schmidt, 2008], the chirp scaling (CS) algorithm [Papoulis, 1968; Raney, 1994; Carrara et al., 1995], and the “wavenumber domain” or “ω-k” algorithm [Rocca, 1987; Cafforio et al., 1991; Bamler, 1992; Carrara et al., 1995]. The ω-k algorithm, which was borrowed by geophysics [Rocca, 1987] and widely adopted by the SAR community [Bamler, 1992; Cumming and Wong, 2004; Reigber, 2006], provides the highest precision among the frequency-domain imaging algorithms, but it needs interpolation to implement Stolt mapping, which mainly determines the precision and efficiency of the algorithm. An approximated but computationally more efficient version of the ω-k algorithm is the Fast ω-k (Fω-k) [Carrara et al., 1995], where the non-linear Stolt interpolation is replaced by a convenient azimuth frequency-dependent phase ramp in the range position domain. The Fω-k algorithm is currently used to operationally process spotlight CSK SAR data: it allows images that comply with processing time and quality requirements to be generated [Caltagirone et al., 2014]. As shown in Cafforio et al. [1991], however, the ω-k algorithm provides an effective solution to the SAR inverse problem when the sensor trajectory is a straight line; satellite-borne sensors do not fit this scheme, and the application of migration techniques in such cases requires a careful analysis. The problem of focusing SAR data acquired along curved orbits has been studied in Cafforio et al. [1991], Reighber et al. [2006] and D’Aria and Monti-Guarnieri [2007]. In Cafforio et al. [1991], an analytical solution to focus SAR data acquired along a circular orbit by means of the ω-k algorithm is presented. A straight line is matched to the circular orbit before applying the ω-k algorithm. Authors state that this approach provides a good solution for short integration times. Our work provides a generalization of the Cafforio’s solution. A modification of the ω-k scheme can be found in Reighber et al. [2006]. However since this method has been developed to focus airborne SAR data characterized by severe orbit irregularities, it has a higher degree of complexity with respect to the algorithm we propose in this paper. In fact, the velocity of a space-borne platform is quite uniform, and a simple focusing scheme can be designed. Our method is simpler and capable to remove the ω-k limitations in focusing CSK spotlight SAR data, characterized by longer integration times.
needed to get sub-metric spatial resolutions. We present a numerical solution to the problem of matching a straight line to the real curved satellite orbit which could be slightly different by a circular one. The parameters of such effective rectilinear orbit are used to focus sub-metric spotlight SAR data by means of the $\omega$-k algorithm. In the following we describe the Fast $\omega$-k and $\omega$-k algorithms, showing how the proposed curved orbit handling allows to properly focus COSMO-SkyMed (CSK) spotlight SAR data. The paper is organized as follows. First the CSK dataset used for evaluation of performance quality is presented and details on the focusing scheme of CSK spotlight SAR data are showed. Then an innovative curved orbit handling, including motion compensation, is proposed and results are presented using both F$k$ and $\omega$-k focusing algorithms. Results have been obtained using CSK SAR data acquired in spotlight sliding mode. Finally, a few conclusions are drawn.

**Dataset description**

The algorithms’ performances have been tested using CSK data. Specifically, we used 81 spotlight level 0B RAW_B data; 49 scenes were acquired in HH polarization and the remaining ones in VV polarization. The incidence angle ranges from 22° to 58°. These data have a standard scene size of 10 km × 10 km and a spatial resolution of 1.0 m × 1.0 m and are single look. The detailed CSK product specifications are available online [Caltagirone et al., 2014]. For testing purposes we selected three sites containing some calibration corner reflectors, specifically Mendoza (Argentina), Metaponto (Italy), and Lodi (Italy). These sites are equipped respectively with 12, 4, and 1 corner reflectors. We have analyzed 50 images over Mendoza, 5 over Metaponto, and 1 over Lodi. We have tried to acquire images with the corner reflectors located near the scenes’ corners, where the focusing performances of the algorithms are usually less effective. The dataset includes also urban areas and sites of special interest where the performances of the proposed algorithm have been assessed by visual inspection of focused images.

**CSK spotlight focusing algorithm approach**

The focusing spotlight SAR data software, currently integrated into COSMO-SkyMED Ground Segment, is described in Italian Space Agency [2001]. It implements a focusing kernel based on a variant of the $\omega$-k algorithm called Fast $\omega$-k [Carrara et al., 1995]. The block diagram of the processing algorithm is represented in Figure 1.

The main focusing steps are:

a) range compression;

b) azimuth spectrum unfolding;

c) fast $\omega$-k.

**Range Compression**

For each azimuth-constant line, the performed operations are:

a) upsampling: a time domain upsampling is performed. The upsampling of factor $N$ depends on the range sampling frequency $f_c$, the swath extent time $T_{sw}$, and the range chirp rate $f_{rg}$;

b) range ramping: this is a multiplication in the fast time domain by the complex conjugate of the chirp signal $c(\tau)$:
c) range compression: this is performed with a convolution in the fast time (or range time) domain $\tau$. The focusing filter is a chirp signal with chirp rate equal to $f_{rg}$:

$$c(\tau) = \exp(j\pi f_{rg} \tau^2)$$  \[2\]
quality; it can be used with small squint angles [Carrara et al., 1995]. This algorithm has been adopted in the CSK Level 1A Products generation; it implements an approximated version of the ideal Stolt interpolation [Carrara et al., 1995]. The solution has been proven to be compatible with the features and quality of performance required by COSMO-SkyMed as defined in Caltagirone [2014]. The following operations are performed:

a) FFT in azimuth;
b) FFT in range;
c) Azimuth matched filtering; it realizes a perfect focusing only in a reference slant range \( r_0 \) (typically the center of the scene);
d) Stolt transformation; it performs the residual range cell migration correction (RCMC) over all the acquired scene. \( F_{\omega-k} \) realizes a first order approximation of the Stolt interpolation, multiplying a simple phase ramp with range lines in the fast time domain (\( \tau \)) - azimuthal wavenumber domain (\( k_x \)); the effect is a shift in the spectral range domain (\( \omega \)) varying with the azimuthal wavenumber axis \( k_x \). The phase ramp operator is:

\[
F_{\omega} (\tau, k_x) = \exp \left\{ -j \omega_s (k_x) \right\} \quad [3]
\]

where \( \omega_s (k_x) \) is the range spectral shift as a function of \( k_x \) [Carrara et al., 1995], and is defined as

\[
\omega_s (k_x) = \frac{c}{2} \sqrt{\frac{k_x^2}{k_{r0}^2} + k_{z0}^2} - \omega_0 \quad [4]
\]

where

\[
\omega_0 = \frac{c}{2} \sqrt{\frac{k_{r0}^2}{k_{r0}^2} + k_{z0}^2} \quad [5]
\]

is the carrier wavenumber, \( k_{r0} \) is the output range central wavenumber, \( k_{z0} \) is the azimuth central wavenumber, and the shift is null for \( k_x = k_{z0} \) (azimuthal central wavenumber). This approximation is valid for very small squint angles [Carrara et al., 1995].

e) IFFT2.

\( \omega-k \)
An higher azimuth geometric resolution can be obtained using the same block diagram of Figure 1, except for the focusing kernel which should be based on the more accurate \( \omega-k \) algorithm. It consists of the following steps:

a) FFT in azimuth;
b) FFT in range;
c) azimuth matched filtering: at this point the exact RCMC is realized by applying the Stolt interpolation transformation [Carrara et al., 1995], which changes the frame of reference from the \( (\omega-k_\omega) \) wavenumber domain in which the signal is represented into the \( (k_r-k_x) \) domain, where \( k_r \) is defined as the range wavenumber axis after Stolt interpolation:
The Stolt transformation is performed by an accurate sinc interpolation. A regular output \( k_r \) grid is defined for the output signal; the grid extension is \( B_{kr} \) and it is centered at \( k_{r0} \):

\[
k_{r0} = \sqrt{\left(\frac{\omega_0}{c / 2}\right)^2 - k_{x0}^2}
\]

with \( k_{x0} \) being the azimuth central wavenumber

\[
B_{kr} = \frac{2\pi}{\Delta r}
\]

where \( \Delta r \) is the output range pixel spacing.

d) the interpolation grid over the \( \omega \) domain is evaluated using the inverse form of [6];

e) Finally the IFFT2 is performed.

Curved orbit handling

In this section we describe a method to handle satellite curved orbit in focusing algorithms. The techniques presented in previous section apply whenever a rectilinear path can be assumed for the sensor; when the sensor moves along a curved path, the hyperbolas generated by point targets vary with ranges differently from the ones that would be experienced with a sensor flying along a straight line. It is possible to modify the curved path so that it coincides, at its best, with a linear one. One possible solution is to achieve the exact correspondence at two predefined range values, so that the error can be kept very small at all ranges of interest. This procedure allows the effects of the earth’s rotation to be compensated too; with reference to Figure 2 we define an effective rectilinear flight path that will be used in the \( \omega-k \) algorithm in order to minimize errors at all range swaths. We define a constant velocity \( v_{\text{eff}} \) that is a relative velocity but takes into account the rectilinear orbit. The range of the closest approach range \( r_0 \) is evaluated starting from this rectilinear orbit, which is not the original one. In the case of a rectilinear orbit and for both squinted and non-squinted acquisition geometry, the Doppler rate \( f_r \) has the simplified form:

\[
f_r = -\frac{2 \cdot \alpha_{\text{eff}}^2}{\lambda \cdot \left(\frac{r_0}{\cos(\phi_0)} + \Delta r\right)} \cdot \cos^2(\phi) \]

where \( \lambda \) is the wavelength, \( \phi \) the squint angle, \( \phi_0 \) the squint angle at near range, \( r_0 \) the closest approach range and \( \Delta r \) the slant range. The true Doppler rate deviates from this expression, but the deviation could also be considered small for large swaths on the ground.
The analytic nonlinear system in [10] could be formulated in order to evaluate $v_{\text{eff}}$ and $r_0$, that is, the effective rectilinear orbit.

$$
\begin{align*}
\frac{f_{R_{1/3}}}{r_0} &= \frac{2 \cdot v_{\text{eff}}^2}{\lambda \cdot \left( \frac{r_0}{\cos(\phi_0)} + \Delta r_{1/3} \right)} \cdot \cos^2(\phi_{1/3}) \\
\frac{f_{R_{2/3}}}{r_0} &= \frac{2 \cdot v_{\text{eff}}^2}{\lambda \cdot \left( \frac{r_0}{\cos(\phi_0)} + \Delta r_{2/3} \right)} \cdot \cos^2(\phi_{2/3}) \\
\frac{f_{d_0}}{r_0} &= -\frac{2}{\lambda} \cdot v_{\text{eff}} \cdot \sin(\phi_{\text{eff}0}) \\
\frac{f_{d_{1/3}}}{r_0} &= -\frac{2}{\lambda} \cdot v_{\text{eff}} \cdot \sin(\phi_{\text{eff}1/3}) \\
\frac{f_{d_{1/2}}}{r_0} &= -\frac{2}{\lambda} \cdot v_{\text{eff}} \cdot \sin(\phi_{\text{eff}1/2}) \\
\frac{f_{d_{2/3}}}{r_0} &= -\frac{2}{\lambda} \cdot v_{\text{eff}} \cdot \sin(\phi_{\text{eff}2/3})
\end{align*}
$$

The first and second equations force the true Doppler rate, evaluated at 1/3 and 2/3 of the range swath (and azimuthal central time), to be equal to those that would be obtained on the effective rectilinear orbit. The other 4 equations force the Doppler centroid evaluated at near range, 1/3, 1/2, and 2/3 of the range swath to be equal to the effective rectilinear orbit ones. The system solution ($v_{\text{eff}}$, $r_0$, $\phi_{\text{eff}0}$, $\phi_{\text{eff}1/3}$, $\phi_{\text{eff}1/2}$ and $\phi_{\text{eff}2/3}$) is obtained by the numerical
Newton method; the group of four parameters $(v_{\text{eff}}, r_0, \phi_{\text{eff}0}, \phi_{\text{eff}1})$ is used in the Fɷ-k and allows CSK spotlight SAR data to be accurately focused in the azimuth direction.

**Motion Compensation**

An hyperbolic range history is assumed in the Fɷ-k focusing algorithm. This approximation becomes less accurate for increasing observation times on the curved orbit. For an observation time of 1.5 to 2 seconds, as is the case of CSK sliding mode, this approximation introduces a cubic-like error (Fig. 3); it increases with the integration time. The effect on target Impulse Response Function (IRF) is asymmetric side lobes and de-focusing, as showed in the following section. Some solutions exist in the literature that use a numerical approach to cope with this problem, for example D’Aria and Monti Guarnieri [2007]. The two step algorithm (TSA) [Fornaro, 1999], is widely used for trajectory deviation compensation (or Motion Compensation MoCo) of airborne SAR. The so-called beam center approximation is needed to allow this efficient MoCo during SAR focusing operation: it assumes motion errors related to all targets within the azimuth beam to be equal to that at the center beam [Pau Prats et al., 2014]. In the spotlight CSK focusing algorithm a numerical approach has also been selected, which resembles the motion compensation approach in airborne systems [Moreira, 1996; Fornaro, 1999]. The novelty of our method consists in introducing an aperture-dependent motion compensation using the scene center as reference point. The concept of aperture-dependent MoCo has been introduced by [Camara-Macedo et al., 2007] to mitigate scene topography and beam-center approximation induced errors, by taking all scene points to run a post-processing step.

![Figure 3 - Cubic-like error due to curved orbit: line-of-sight (LOS) approximation error computed for a reference target in the middle of the scene and extrapolated to the full scene.](image)

The line-of-sight (LOS) approximation error $\delta r_{\text{hyp}}$ shown in Figure 3, is computed for a reference target $P_{\text{ref}}$, located in the middle of the scene, as the difference between the
effective rectilinear and real orbit LOS’ along the whole data take acquisition interval. The approximation error $\delta r_{hyp}$ is transformed into a phase term used to mitigate the curved orbit effect by means of the following multiplicative term:

$$M(t, f_r; r_{ref}) = \exp \left[ j \cdot \frac{4\pi}{c} \cdot (f_0 + f_r) \cdot \delta r_{hyp} (t; r_{ref}) \right]$$

where $r_{ref}$ is the reference range (set to the midrange), $f_0$ the frequency carrier, $t$ the ‘slow time’, $f_r$ are the range frequencies (chirp frequencies). Such a phase term shifts the signal both in terms of envelope in range direction and phase in azimuth direction (with fixed $r_{ref}$), correcting the non-hyperbolic term. The correction we introduce in this paper is only valid for a target located on the center of the scene. However, a polynomial extrapolation applied to all points located at the same range bin introduces an acceptable small phase error for the CSK sliding spotlight acquisition mode. This error might not be small for more demanding scenarios (better resolution, larger coverage and squinted acquisitions). With such a step, a pure hyperbolic phase history is forced, so that a conventional hyperbolic kernel can still be used to process the data without modifications. The correction is realized in azimuth-time. It is worth noting that this correction is only valid for the mid-range. Therefore, a second-order compensation might be necessary, as is usually the case with airborne systems. However, it turns out that due to the large separation between the sensor and the scene and the small swath width, the range-dependency can be neglected.

Results on real CSK data

In this section we present some results obtained by applying the curved orbit handling method to the CSK dataset described in section 2. Figures 4 and 5 report CSK spotlight SAR data acquired on Metaponto and Lodi sites; they show that if real orbit parameters are used instead of effective ones, de-focusing occurs.

Concerning Motion Compensation, Figures 6-8 show the azimuth IRF profile and the target IRF contour of corner reflectors in CSK spotlight data take, at scene center (Metaponto and Lodi site) and scene border (Mendoza site) respectively, with and without Motion
Compensation. It is evident as the side lobes asymmetry (at scene center) and the de-focusing (at scene border) are mitigated using proposed Motion Compensation. Tables 1-3 provide a quantitative proof of side lobes asymmetry (PSLR measurement) and de-focusing (IRF shape measurement) mitigation. Figures 6-8 shows the phase of two-dimensional FFT; the curvature of phase fringes when MoCo is not applied indicates a lack of linearity of azimuth phase and they are related to azimuth IRF shape distortions [Carrara et al., 1995]. Using the proposed MoCo, the fringes curvature is removed.

Figure 5 - Lodi CSK spotlight data take processed using Fɷ-k kernel; real curved orbit (left) and effective rectilinear curved orbit (right) have been used.

Figure 6 - Metaponto (Italy) CSK spotlight data take: Azimuth IRF profile (not hamming weighting is applied), target IRF contour and phase of two-dimensional FFT of a corner reflector located at scene center without MoCo (top) and with MoCo (bottom).
Table 1 - Metaponto (Italy): Measured corner reflectors azimuth PSLR (not hamming weighting is applied) and IRF Shape with and without MoCo (center scene).

<table>
<thead>
<tr>
<th>Without Motion Compensation</th>
<th>With Motion Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth ratio -6dB/-3dB = 1.366</td>
<td>Azimuth ratio -6dB/-3dB = 1.364</td>
</tr>
<tr>
<td>Azimuth ratio -10dB/-3dB = 1.677</td>
<td>Azimuth ratio -10dB/-3dB = 1.672</td>
</tr>
</tbody>
</table>

Figure 7 - Lodi (Italy) CSK spotlight data take: Azimuth IRF profile (not hamming weighting is applied), target IRF contour and phase of two-dimensional FFT of a corner reflector located at scene center, without MoCo (top) and with MoCo (bottom).
Table 2 - Lodi (Italy): Measured corner reflector Azimuth PSLR (not hamming weighting is applied) and IRF Shape with and without MoCo (scene center).

<table>
<thead>
<tr>
<th>Without Motion Compensation</th>
<th>With Motion Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth PSLR sx [dB] = -12.03</td>
<td>Azimuth PSLR sx [dB] = -12.55</td>
</tr>
<tr>
<td>Azimuth PSLR dx [dB] = -14.35</td>
<td>Azimuth PSLR dx [dB] = -13.64</td>
</tr>
<tr>
<td>Azimuth ratio -6dB/-3dB = 1.363</td>
<td>Azimuth ratio -6dB/-3dB = 1.362</td>
</tr>
<tr>
<td>Azimuth ratio -10dB/-3dB = 1.671</td>
<td>Azimuth ratio -10dB/-3dB = 1.667</td>
</tr>
</tbody>
</table>

Figure 8 - Mendoza (Argentine) CSK spotlight data take: Azimuth IRF profile (not hamming weighting is applied), target IRF contour and phase of two-dimensional FFT of a corner reflector located close to border scene, without MoCo (top) and with MoCo (bottom).
The large availability of data on calibration sites has allowed us to effectively assess the performances of the proposed methodology on a wide range of beams and polarizations. The whole dataset has been focused with both the \( \omega-k \) and the \( F_\omega-k \) algorithms, generating SCS_B products. On corner reflectors deployed on the scenes, the IRF has been measured [Marelli, 1980; Martinez and Marchand, 1993]; these quality measurements allow to quantitatively assess the quality of SAR images obtained with the proposed orbit curved handling algorithm and the described focusing kernels. In addition, considerations are made in terms of data interpretation of natural targets.

**IRF Measurements on Single Point Target**

In this section we test the algorithm performances on several CSK data takes to demonstrate reliability of the proposed method. Table 4 summarizes the results of the analysis for some of CSK data takes. Mendoza test site has been chosen because it is equipped with a large number of corner reflectors (12), deployed on a large portion of the scene (some corner reflectors are located near the scene borders). The second satellite has been chosen to show azimuth resolution performance at different beams and polarizations. A beam with small look angle has been chosen to assess results for different satellites. Both \( \omega-k \) and \( F_\omega-k \) algorithms have been used to generate SCS_B products.

**Table 4. Measured Resolutions on Mendoza test site.**

<table>
<thead>
<tr>
<th>Image ID</th>
<th>( \omega-k ) Mean azimuth resolution</th>
<th>Fast ( \omega-k ) Mean azimuth resolution</th>
<th>( \omega-k ) Worst azimuth resolution</th>
<th>Fast ( \omega-k ) Worst azimuth resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSKS1_SCS_B_S2_04_VV_RA_SF_20120728110755</td>
<td>0.874</td>
<td>1.001</td>
<td>0.878</td>
<td>1.030</td>
</tr>
<tr>
<td>CSKS2_SCS_B_S2_04_HH_RA_SF_20120415110852</td>
<td>0.864</td>
<td>0.989</td>
<td>0.870</td>
<td>0.999</td>
</tr>
<tr>
<td>CSKS2_SCS_B_S2_12_HH_RA_SF_20120629111410</td>
<td>0.868</td>
<td>0.956</td>
<td>0.872</td>
<td>0.967</td>
</tr>
<tr>
<td>CSKS2_SCS_B_S2_19_VV_RA_SF_20120726111953</td>
<td>0.876</td>
<td>0.968</td>
<td>0.878</td>
<td>0.980</td>
</tr>
<tr>
<td>CSKS2_SCS_B_S2_33_HH_RA_SF_20120529113222</td>
<td>0.895</td>
<td>1.020</td>
<td>0.902</td>
<td>1.031</td>
</tr>
<tr>
<td>CSKS3_SCS_B_S2_04_HH_RA_SF_20120619110814</td>
<td>0.872</td>
<td>0.991</td>
<td>0.874</td>
<td>1.008</td>
</tr>
<tr>
<td>CSKS4_SCS_B_S2_04_HH_RA_SF_20120622110815</td>
<td>0.864</td>
<td>0.988</td>
<td>0.869</td>
<td>1.007</td>
</tr>
</tbody>
</table>
Figure 9 displays an image acquired over Mendoza and the position of the corner reflector n° 6. The azimuth cuts of this target, obtained with Fω-k and ω-k, have been plotted in Figure 10. As expected, there is an improvement of the azimuth geometric resolution when using ω-k kernel.

![Corner reflector 6](image)

**Figure 9 - Mendoza (Argentine) - “COSMO-SkyMed Product - ©ASI - Agenzia Spaziale Italiana - (2013). All Rights Reserved”.

![Azimuth Profile](image)

**Figure 10 - Azimuth cuts obtained with Fast ω-k (solid line) and ω-k (dashed line) on a Mendoza corner reflector near scene border.**
Figure 11 - Azimuth resolution vs Beam ID and CSK Satellite ID.
A larger data take (50 acquisitions on Mendoza) has been considered to graphically display (Fig. 11) the azimuth geometric resolution measurements in terms of Beam and satellite ID. Figure 11 shows trends of azimuth geometric resolution versus Beam ID obtained with $\omega$-k and $F_{\omega}$-k focusing kernel; each graph is relative to a different satellite and both ascending and descending orbit directions are considered. It is worth noting that a different approach has been used (in $\omega$-k processing) with respect to the CSK operative spotlight processor ($F_{\omega}$-k). The processed azimuth bandwidth is automatically evaluated using sensor, geometry and doppler parameters [Lanari et al., 2001; Lanari et al., 2003] in order to use always the antenna aperture beamwidth value associated to the half power of -3 dB relative to the peak of the CSK antenna pattern. In this way we can experimentally obtain better overall IRF quality performances for each scene. Using this approach for $\omega$-k processing, Figure 11 shows that azimuth resolution worsens while Beam ID increases, for azimuth bandwidth decreases from 8600 to 8300 Hz on Mendoza site. On the other hand, $F_{\omega}$-k processing had been performed with fixed azimuth bandwidth; in this case the best azimuth performances are obtained for Beam IDs close to the corner reflectors boresight corresponding to 10 and 11 Beam IDs. As expected $\omega$-k generates products with better (up to 12%) azimuth geometric resolution than $F_{\omega}$-k focusing kernel. Figure 12 is a synoptic of Mendoza site azimuth geometric resolution analysis, considering all 50 used data takes. Previous results are confirmed, and a further element can be read by the graph: for each Beam ID, the $F_{\omega}$-k produces a higher dispersion of azimuth performances than the $\omega$-k focusing kernel. This is related to non-uniform performances of the $F_{\omega}$-k algorithm, depending on the position of corner reflectors in the data take; in particular point target IRF measurements worsen from center to the border of the scene. On the other hand, low dispersion of measurements obtained with $\omega$-k algorithm shows that all CSK satellites ensure similar IRF performances for each Beam ID, orbit direction and polarization.

![Figure 12 - Azimuth geometric dispersion with different satellite, beam ID, orbit direction and polarization: $F_{\omega}$-k (up) measures vs $\omega$-k (down) one.](image)

In the following, also measurements on Metaponto (Tab. 5) and Lodi (Tab. 6) test sites are presented. Lodi and Metaponto sites show azimuth resolutions better than Mendoza, due to an higher acquired azimuth bandwidth (evaluated on the -3dB aperture of the azimuth antenna pattern peak).
The analysis of all performed measurements demonstrates the effectiveness to focus CSK SAR images using the proposed curved orbit handling method with both Fω-k and ω-k focusing algorithm. As expected, greater improvements, up to 12%, of the azimuth geometric resolution, with respect to the CSK standard SCS_B products [Caltagirone, 2014], is obtained if the ω-k algorithm is used. This improvement does not affect the value of all the other quality parameters which appear to be in line with CSK mission requirements.

**Qualitative analysis on natural scatters**

A qualitative analysis in terms of two-dimensional IRF and ability to discriminate two close objects have been performed on Bari (Fig. 13 and 14) and Metaponto (Fig. 15 and 16) data takes.
Figure 13 shows the detail of two nearby strong scatters of Bari data take; in Figure 14, the comparison of the focusing performances on the scatters is reported: it is evident the cloverleaf effect due to the not perfectly corrected Doppler history range migration in Fω-k (left side of the image); on the other hand a better energy focusing of scatters is obtained with ω-k.

![Figure 14 - Targets of opportunity in Bari data take: IRFs obtained with Fω-k (left) and standard ω-k (right).](image)

Figure 15 displays an image acquired over Metaponto test site, the position of the corner reflectors no. 1 and 3, and the position of natural high intensity scatters. In Figure 16, natural scatters are over-sampled in order to highlight the improvement of the discrimination capability obtained on CSK spotlight products with enhanced geometric azimuth resolution (image b in Fig. 16) compared to standard products (image a in Fig. 16).

![Figure 15 - Metaponto (Italy) - “COSMO-SkyMed Product - ©ASI - Agenzia Spaziale Italiana - (2009). All Rights Reserved”.](image)
It follows that it is possible to generate CSK spotlight products such that the ability to discriminate two close objects gets better without affecting the performance of geometric qualities in other ways.

Figure 16 - Some opportunity targets present in the Metaponto test site (left). Detail of the SAR images processed with exact $\omega$-k (b) and $F_\omega$-k (a) - “COSMO-SkyMed Product - ©ASI - Agenzia Spaziale Italiana - (2009). All Rights Reserved”.

Figure 17 shows that, despite the use of the precise Stolt interpolation, a residual migration of Doppler history remains. This migration appears to be small and no longer quadratic: the secondary cell migration has been removed, together with a part of the linear one (in fact, the slope is reduced). This anomaly is present in all the analyzed CSK data.

Figure 17 - Doppler history in range Doppler domain, after focusing with $F_\omega$-k (left) and exact $\omega$-k(right).
Conclusion
In this paper, a new curved orbit handling and Motion Compensation refinement methodology has been described, that allows to use both $\omega$-k and $F_{\omega}$-k algorithms to focus CSK data acquired along curved orbits. Several CSK spotlight raw data, acquired on various sites, with different look angle, orbit direction, satellite ID, polarization have been processed using the proposed methodology with both $\omega$-k and $F_{\omega}$-k algorithm, generating Single Look Complex Slant and Hamming-weighted (SCS_B) products. Each product has been analyzed in terms of IRF evaluated at corner reflectors deployed in the scene. Results demonstrate the effectiveness of the proposed methodology to focus CSK spotlight SAR data. Respectively, sub-metric and metric azimuth spatial resolution can be obtained when the $\omega$-k algorithm, and its approximation $F_{\omega}$-k are used. Despite the use of the accurate $\omega$-k, a residual migration of Doppler history is still present. This anomaly has been observed in all the analyzed CSK data. The understanding of the cause of this residual phase history migration and its removal deserves further efforts since it could improve the CSK spotlight product quality.

References


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